Determining a model to predict the dynamic stretch of a bungee cord
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Introduction

We have set about the goal of developing a bungee jump system that will provide an ideal experience for a falling egg by maximizing the passenger’s free fall time, minimizing the force it experiences during deceleration, and not allowing the passenger to hit the ground. In order to develop an ideal bungee system, we must first determine the properties of our bungee cord. Under the assumption that our cord behaves similarly to a spring, we have begun to analyze the properties of our cord in terms of Hooke’s law. According to Hooke’s law, the stretch $x$ of an ideal spring is proportionally related to the restoring force $F$ that acts against the pulling force of a weight attached to a hanging spring or cord. For an ideal spring, these variables are related by a spring constant $k$ in the following equation:

$$F_{spring} = -kx$$  \hspace{1cm} (1)

The term for force in this equation represents the restoring force, and it acts in the opposing direction of the stretch of the spring to pull the spring back towards its resting equilibrium position. Since we are trying to characterize a cord system, Hooke’s law will be interpreted as the restoring force of the cord set equal to the spring constant and stretch displacement of the cord. For a hanging cord, the magnitude of this force is equal to the magnitude of the weight that pulls the cord downward when the cord has reached its full static stretch. While the spring constant should remain constant for an ideal bungee cord, our previous static experiment found the spring constant to vary with cord length. In fact, we found that the spring constant $k$ was related to the length $l$ of the cord by the following exponential equation:

$$k = 9.3179e^{-1.578l}$$  \hspace{1cm} (2)

According to this equation, the spring constant of the cord decreases as the cord’s length is increased. Therefore, we found that our cord did not ideally model Hooke’s law since our spring constant was not actually found to be constant. Rather, it exhibited an inverse relationship with the length of the cord. Despite the fact that the cord’s spring constant did not exhibit the ideal behavior of Hooke’s law, this model for $k$ provides us with a prediction for the value of the spring constant when different lengths of the cord are used. Furthermore, having a model for the spring constant allows us to apply the predicted value for this constant to further analyses. Using this finding, we decided to analyze the cord in terms of energy conservation. The potential energy $PE$ of a spring is related to the stretch of the spring by the following equation:

$$PE = \frac{1}{2}kx^2$$  \hspace{1cm} (3)

This equation can be used when considering the conservation of energy of a spring or bungee system. In our bungee system, when a passenger is at the top of the drop, the
passenger has a maximum potential energy. After ‘jumping’ from a height $h$ and reaching the bottom of the jump, the passenger loses potential energy that is transferred to the cord as it stretches a distance $x$. Therefore, the energy of the system is conserved as a result of an energy transfer that occurs between the passenger and the cord as the weight of the falling passenger stretches the cord beyond its static equilibrium position. Because the potential energy is also equal to the height $h$ of the passenger at the top of the jump multiplied by the weight of the passenger, the potential energy that is transferred to the cord can be written according to the following equation:

$$mgh = \frac{1}{2} kx^2$$  \hspace{1cm} (4)

In order to express the potential energy of the cord in terms of our previous findings, we inserted Equation 2 for $k$ in the following conservation of energy equation:

$$mgh = \int kxdx$$  \hspace{1cm} (5)

Because $k$ varies with the length of the cord that is used, Equation 5 contains an integral expressing the potential energy of the spring as the length of the cord is manipulated. Equation 5 was integrated with the input of Equation 2 for the value of $k$ using Mathematica to provide the following final equation:

$$mgh = e^{-1.578}(-3.742 - 5.905)$$  \hspace{1cm} (6)

With Equation 6, we can predict the distance $x$ that a cord will stretch during a bungee jump when a given static stretch length $h$ of cord is used with a known weight attached to the end of the cord as the bungee passenger. The static stretch in this equation serves as the height that the passenger will fall from. Therefore, the static stretch $h$ can be used to predict the dynamic stretch $x$ of the cord. This study aimed to determine whether or not this model serves as an accurate predictor for the stretch of the cord during a dynamic bungee jump.

**Methods**

A dynamic experiment was conducted in order to analyze the stretch distance of our bungee cord from its static equilibrium position when an attached weight was allowed to free fall from the top of the bungee system and stretch the cord past its weighted static resting position. A metal weight was attached to the unsuspended end of the cord, with this end then dropped from the top of the bungee system for each trial. The mass of the attached metal weight was held constant throughout the experiment, with an attached mass of 0.13 kg used for each trial of the study. A tape measurer was hung from the top of the bungee system in order to allow for measurements of the length of the cord. Five different cord lengths were used for the experiment, including 0.490, 0.565, 0.600, 0.635, and 0.670 m lengths. This un-stretched length for each cord was measured before any weights were added to the hanging end of the cord. Another measurement was made for the static stretch of the cord when the weight was added to the end of the cord, with this stretch length and the original un-stretched length of the cord added together to
provide the total height $h$ of the cord system. After measuring these static stretches, we then conducted a dynamic experiment in order to determine how far the cord would stretch when the attached mass was allowed to free fall from the maximum height of the system. Five bungee jump trials were conducted for each cord length and a video camera was used to record the bungee jump and visualize the location of the cord’s stretch that resulted from the bungee jump. When the passenger was dropped from the top of the system, the video camera began recording. The video footage was then analyzed to determine the location of the passenger when the cord had reached its maximum stretch. Measurements were taken by measuring the distance on the tape measurer from the top of the bungee cord to the location of the top of the weighted hanger that served as the passenger. The values of the stretched length of the cord for each trial were then averaged to provide the average dynamic stretch length for each cord length that was used. The static stretch length $h$ was subtracted from the total dynamic stretch length in order to provide the value $x$ for the displacement of the dynamic stretch from the static stretch position. These values represented the realized dynamic stretch of the cord, while the predicted values for $x$ given the static stretch $h$ were also calculated using Equation 6.

**Figure 1. Experimental Design.** The bungee cord was attached to a fixed bar at the top of the bungee system, with a weight then attached to the hanging end of the cord. The static length of the cord with the weight attached is represented by $h$ in the diagram above, with $x$ representing the dynamic displacement in the length of the cord from the static equilibrium position. A tape measurer was attached to the same bar that the cord was attached to in order to allow for the measurement of the length of the cord based on the position of the top of the weighted hanger in relation to the top of the bungee system. A video camera was used to record the bungee jump and visualize the measurements for the dynamic stretch of the cord. The diagram on the left represents the static stretch system in which the cord was allowed to come to a resting position after the weighted hanger was attached. The diagram on the right demonstrates the dynamic stretch system in which the weighted hanger was allowed to free fall from the top of the system, causing the cord to stretch a distance $x$ from its initial static equilibrium position.
Results

Because five different lengths of cord were used for experimentation, five different static stretch lengths were obtained from attaching the weighted hanger to the end of the bungee cord and allowing the cord to stretch downward. Using Equation 6, the displacement, or difference in the length of the bungee cord stretch between the dynamic and static stretches, was calculated using the known static stretch lengths \( h \) of the bungee cord as a predicted value for the displacement \( x \). The equation obtained from plotting the predicted dynamic displacement versus the static stretch length yielded the equation \( y = 0.0678x + 0.6369 \), while the equation obtained for the actual experimental displacement values was \( y = 0.8886x + 0.071 \). Because a discrepancy existed between the equations for the predicted and realized values of the dynamic displacement, the actual stretch displacement values were plotted against the predicted displacement values in order to allow for the construction of an adjustment model between predicted and realized values. The adjustment model provided the equation \( y = 13.024x - 8.2187 \), where \( x \) represents the predicted values and \( y \) represents the actual values that should theoretically be obtained for the dynamic displacement. Table 1 below provides the data for the initial measured static stretch of the system and the corresponding values for the realized and predicted dynamic displacements.

Table 1. Static stretch length \( h \), average realized kinetic displacement \( x \), and predicted displacement. The table provides data for the displacement \( x \) of the cord from its static stretch position \( h \) when the weighted hanger was allowed to fall downward and pull the cord to a maximum stretch length. The realized displacement was taken as the average stretch from the five trials conducted at each cord length.

<table>
<thead>
<tr>
<th>Static Stretch Length ( h ) (m, ± 0.001 m)</th>
<th>Average Realized Dynamic Displacement ( x ) (m, ± 0.001 m)</th>
<th>Predicted Displacement ( x ) (m, ± 0.001 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.790</td>
<td>0.770</td>
<td>0.690</td>
</tr>
<tr>
<td>0.920</td>
<td>0.888</td>
<td>0.700</td>
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<td>0.708</td>
</tr>
<tr>
<td>1.110</td>
<td>1.034</td>
<td>0.712</td>
</tr>
</tbody>
</table>

Table 1 provides data for the static stretch of the bungee cord with the weighted hanger attached and the corresponding values for the displacement from this position due to the dynamic stretch of the system. The predicted values for this displacement were calculated using Equation 6 prior to experimentation, with the realized values then recorded as the average displacement for the five trials conducted at each given static stretch length. The realized values were recorded as the difference between the length of the maximum dynamic stretch of the cord and the static stretch length. The raw uncertainty for each measurement was ± 0.001 m. After obtaining these values, the predicted and actual values of the dynamic stretch displacement were plotted as functions of the initial static stretch length for each trial.
Figure 2. Predicted dynamic displacement vs. static stretch length. The graph provides a linear relationship between the predicted dynamic displacement of the bungee cord from its static equilibrium position and a known static stretch value. The graph was obtained by plotting data from Table 1.

Figure 2 provides a linearized graph of the predicted dynamic displacement of the cord at five different static stretch lengths. The predicted values were determined by inserting the static stretch length $h$ into Equation 6 and solving for the dynamic displacement $x$ from this static position. An equation of $y = 0.0678x + 0.6369$ was obtained for the relationship between the predicted displacement and the static stretch length, with $x$ representing the static stretch length and $y$ representing the dynamic displacement. The raw uncertainty for the all measurements was ±0.001 m and an $R^2$ value of 0.9964 was obtained for the plot of this relationship.
Figure 3. **Average realized dynamic displacement vs. static stretch length.** The graph provides a linear equation relating the realized dynamic displacement to the initial static stretch value. The graph was obtained by plotting data from Table 1.

Figure 3 provides a linearized graph of the averaged realized dynamic displacement of the cord versus the static stretch length of the cord for five different lengths. The equation \( y = 0.8886x + 0.071 \) was obtained for this relationship with \( x \) representing the static stretch length and \( y \) representing the dynamic displacement. An \( R^2 \) value of 0.96473 was obtained for the plot of this relationship, with \( \pm 0.001 \) m serving as the raw uncertainty for all measurements. Because the graphs for our predicted and realized values resulted in different models for the dynamic stretch of the cord, we plotted the actual dynamic displacement values versus the predicted displacement values in order to obtain an equation that would allow for the adjustment of the predicted displacement values to provide a closer estimate of the actual values for the dynamic stretch.
Figure 4. Average realized displacement vs. predicted dynamic displacement from static resting position. The graph provides an equation relating the predicted displacement to the actual displacement of the cord under dynamic conditions.

Figure 4 provides a graph of the linear relationship between the predicted dynamic displacement and the actual displacement. The equation $y = 13.024x - 8.2187$ was obtained for this graph, with $x$ representing the predicted displacement and $y$ representing the actual dynamic displacement of the cord from the static stretch position. An $R^2$ value of 0.95494 was obtained for this relationship. This equation serves as a correction function to adjust the predicted values for displacement so that they more precisely estimate the actual values for the cord’s stretch from its equilibrium position in a dynamic system.

### Discussion

We found that our predicted model for the dynamic displacement of the bungee cord from the static stretch position did not provide the same values for the displacement as those that were obtained from the actual measurements of the displacement. Therefore, in order to correct for this discrepancy between the predicted and realized models for displacement, we plotted the realized displacement values versus the predicted displacement values to provide an equation that will allow us to use our initial prediction value to make an accurate prediction of the realized displacement. Our corrected predictions will made by inserting the static stretch $h$ of the cord into Equation 6 and solving for $x$ and then inserting this predicted value in for $x$ in our adjustment equation. By using this adjustment equation, we will be able to correct our initial predicted displacement values to more precisely predict the actual dynamic stretch of the bungee cord. Overall, the adjustment equation that we obtained from this study will be used in
order to design an ideal bungee system for an egg drop. Using this adjusted model, we will be able to predict how far the cord will stretch given the initial length of the cord and the static stretch length of the cord when a weighted mass such as an egg is attached to the cord’s hanging end. Our previous finding of an equation that relates the spring constant to the original length of the cord will allow us to predict the static stretch of the cord when a certain length is used.

Sources of error were also present in our experiment. For example, the capturing speed of the video camera that we used to record and visualize the dynamic stretch distance only allowed us to view images of the video at intervals of 0.1 seconds. Therefore, we may not have been able to visualize the exact moment when the cord reached its maximum stretch if this occurred during a time that could not be visualized by a 0.1 second interval. If this were the case, then a measurement error of more than ±0.001 m could have been made for measurements of the dynamic stretch length. We could correct this error by using a video camera that captures images in lower time intervals. The cord could also have been stretched beyond its elastic range as a result of the repetitive dynamic movement of the weight causing the cord to exceed its elastic stretch limits. This stretching would mean that the static stretch length of the bungee cord would vary across trials. We could have measured the static stretch after each dynamic trial in order to determine if the cord had in fact stretched to a longer starting length after each bungee jump. Overall, we were able to utilize the equation that we had previously obtained for our spring constant as function of cord length to create an energy model for predicting the dynamic stretch of the bungee cord when a weighted mass is attached to its end. Because this model did not accurately predict the actual dynamic stretch of the cord, an adjustment model was made that will allow us to correct our predicted value to a more precise estimation of how far the bungee cord will stretch from the static stretch position under the dynamic conditions of a bungee jump. We now have an equation that allows us to determine the spring constant of the cord given its starting length, as well as an equation that allows us to determine how far the cord will stretch during a bungee jump.

**Conclusion**

In this study we found a discrepancy between our predicted values for the dynamic stretch and the actual dynamic stretch of the bungee cord at given starting static stretch lengths. Because of this discrepancy, it was necessary to construct a new model that adjusts for the differences in values obtained from our prediction model and the actual model of the system. We plotted our predicted values versus our realized values for the dynamic stretch, allowing us to obtain an equation that allows us to adjust our predicted values so as to make more accurate predictions. In a future experiment, we will test the effectiveness of this new adjusted model with its use in designing an ideal bungee jump for an egg passenger. Along with the stretch of the cord, we will also need to consider the magnitude of the force that our passenger experiences during acceleration so that we can keep the jump within comfortable limits. We will now be able to design a system that maximizes the stretch of the cord without allowing the egg to hit the ground.

**Pledge:** On my honor, I have neither given nor received any unacknowledged aid on this report.
Jake Roberts