How are the Spring Constant and Height Related?

Introduction

In our second bungee experiment, we again explored relationships between the different variables, such as mass, height, and length of string. This time we used the CWE theorem to observe the relationship between $k$ and height, in particular.

When dropping the egg from the Great Hall, the mass, “m” will initially free fall from a height “h” with gravity, “g”, acting upon it.

$$PE = mgh$$

Then at the bottom of the fall the string will stretch similar to a spring. Here we use the kinetic energy formula for a spring. We also looked at the relationship between

$$KE = \frac{1}{2}kx^2$$

the spring constant, $k$, and the length of the strength. We combined these two equations using the CWE theorem. The CWE theorem states that potential energy is equal to kinetic energy.

$$mgh = \frac{1}{2}kx^2$$

The purpose of this experiment was to build upon our observations from the first bungee lab where we found relationships between variables in Hooke’s Law.

Methods

We carried out this experiment by first deciding to keep mass constant and to perform a dynamic experiment. We chose a mass of about 140 g to model the egg
and harness. We tied our bungee string to the top of the bar that was attached to our lab table. We also attached the force meter to measure the force of the drop through *Capstone*.

![Diagram of the setup](image)

We wanted to make sure our trials did not exceed the force required.

\[ F < 3mg \]  \hspace{1cm} (4)

We used a 50 g hook and attached disks with tape that totaled to about 140 g. We measured the mass to ensure accuracy. The mass with the tape was 0.14066 kg. We made four different knots in the string and set up a tape measure parallel to the string to measure the length of the knots and the displacements. We started by hooking the mass into the first knot and lifting it to the top, where the string was tied, and then we dropped the hook trying not to exert any extra force. We used the CMV app on the iPad to record each drop and play it back in slow motion. This
helped us get more accurate measurements of displacement. We recorded the results and repeated this process for the next three lengths.

We performed a second experiment keeping the mass and length constant. We varied the height of the drop. Instead of lifting the mass to the place of string’s attachment each time, we dropped it from consecutively lower heights. We proposed that keeping all but one variable constant would help us get clearer results. Our set up was the same as above except we only tied one knot at the chosen length. We chose a length that would allow us to perform several drops without the mass hitting the floor. We lifted the mass to the first chosen height and dropped it. We used the CMV app to measure displacement and the force meter to measure the force of the drop. We recorded the results, and we repeated this process at the next six chosen heights.

Results

The main results of the first part of the experiment were that $k$ and $h$ began to show an exponential relationship in the graph of our data. We also graphed several other variables to look at their relationships to one another.

<table>
<thead>
<tr>
<th>L (m)</th>
<th>Mass (kg)</th>
<th>h (m)</th>
<th>x (m)</th>
<th>F (N)</th>
<th>g</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.373</td>
<td>0.14066</td>
<td>1.007</td>
<td>0.634</td>
<td>-4.18</td>
<td>9.81</td>
<td>6.913859836</td>
</tr>
<tr>
<td>0.518</td>
<td>0.14066</td>
<td>1.374</td>
<td>0.856</td>
<td>-4.07</td>
<td>9.81</td>
<td>5.174981713</td>
</tr>
<tr>
<td>0.698</td>
<td>0.14066</td>
<td>1.845</td>
<td>1.147</td>
<td>-4.07</td>
<td>9.81</td>
<td>3.870251172</td>
</tr>
<tr>
<td>0.807</td>
<td>0.14066</td>
<td>2.007</td>
<td>1.2</td>
<td>-4.08</td>
<td>9.81</td>
<td>3.846400448</td>
</tr>
</tbody>
</table>

Figure 1: Data From Part One of the Experiment. This is the data from the experiment where the length and height varied but mass stayed the same.
$X$ was found by subtracting the measured length of the string from the measured height of the drop. The force meter measured $F$, and $k$ was found using Equation 3. The maximum force in this part of the experiment was 4.14 N. Height was measured and .223 m was subtracted from the measured value to account for the tape measurer hanging higher than the actual drop because of the force meter set up. The raw uncertainty of length was +/- 4.24 mm. The raw uncertainty in the measurement of height was +/- 4.24 mm. The uncertainty of displacement was +/- 4.24 mm. The raw uncertainty of mass was +/- .001 g. The raw uncertainty of force was +/- .01 N.

![Graph of Displacement vs. Spring Constant](image)

Figure 2: Graph of Displacement vs. Spring Constant. This graph shows that $x$ and $k$ have an inverse linear relationship.

As the displacement decreases, the spring constant increases. This is an important relationship in Hooke’s Law and the CWE theorem.
Figure 3: Graph of Length vs. Height. This graph shows that length and height has a direct relationship.

As length increases, height also increases. This data is significant for determining the length of our string in relationship to the height of our drop. We could extrapolate this data for a height of 8 meters and find the corresponding length of string.
Figure 4: Graph of Spring Constant vs. Height. The graph shows an exponential relationship between $k$ and $h$.

This graph had the most significance in this part of our experiment. We noticed that the data looked similar to an exponential curve, but we needed to do another experiment to look further.

<table>
<thead>
<tr>
<th>L (m)</th>
<th>m (kg)</th>
<th>h (m)</th>
<th>x (m)</th>
<th>F (N)</th>
<th>g</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.713</td>
<td>0.1418</td>
<td>2.094</td>
<td>1.381</td>
<td>-3.92</td>
<td>9.81</td>
<td>3.054671789</td>
</tr>
<tr>
<td>0.713</td>
<td>0.1418</td>
<td>2.088</td>
<td>1.375</td>
<td>-3.84</td>
<td>9.81</td>
<td>3.072559713</td>
</tr>
<tr>
<td>0.713</td>
<td>0.1418</td>
<td>1.938</td>
<td>1.225</td>
<td>-3.61</td>
<td>9.81</td>
<td>3.59299679</td>
</tr>
<tr>
<td>0.713</td>
<td>0.1418</td>
<td>1.816</td>
<td>1.103</td>
<td>-3.5</td>
<td>9.81</td>
<td>4.152790795</td>
</tr>
<tr>
<td>0.713</td>
<td>0.1418</td>
<td>1.59</td>
<td>0.877</td>
<td>-3.12</td>
<td>9.81</td>
<td>5.751394682</td>
</tr>
<tr>
<td>0.713</td>
<td>0.1418</td>
<td>1.495</td>
<td>0.782</td>
<td>-3.03</td>
<td>9.81</td>
<td>6.801472093</td>
</tr>
<tr>
<td>0.713</td>
<td>0.1418</td>
<td>1.398</td>
<td>0.685</td>
<td>-2.68</td>
<td>9.81</td>
<td>8.288983255</td>
</tr>
</tbody>
</table>

Figure 5: Data From the Second Part of the Experiment. This is the data from the experiment where kept the length constant but changed the height of the drop.

$X$ was again found by subtracting the length from the height. The height of the drop began at 2.094 and decreased in varying increments. The maximum force in this part of the experiment is 4.17 N. The raw uncertainty of length was +/- 3 mm. The raw uncertainty in the measurement of height was +/- 4.24 mm. The uncertainty of displacement was +/- 4.24 mm. The raw uncertainty of mass was +/- .001 g. The raw uncertainty of force was +/- .01 N.
Figure 6: Graph of Height vs. Spring Constant. We used this data to further explore the relationship between \( h \) and \( k \).

By using the exponential equation, we can extrapolate the data for the 8 meter height of the egg drop.

Figure 7: Graph of Height vs. Displacement. This graph shows a linear relationship between height and displacement.
This is relevant because it will give us insight into the amount of displacement the egg will have from a given height. Using this data we can determine how close our egg can get to the ground without breaking.

Figure 8: Graph of Displacement vs. Spring Constant. $X$ and $k$ have an exponential relationship similar to the one of $h$ and $x$.

This graph could be used to verify a spring constant found using Figure 6.

Discussion

Our results from the second part of the experiment support our hypothesis that height and the spring constant have an exponential relationship. Our results seem to align with the CWE theorem as well. The uncertainties were relatively small and are mostly raw data. Displacement had a higher uncertainty because it was found using the difference between two measured values. Other values that had values subtracted from them also had a higher uncertainty than the raw uncertainties, but most of the calculations were simple differences. Therefore, our
results seem reasonable. Only one trial exceeded the maximum force. It exceeded by .04 N.

There could have been error in measurement, error in the force meter and Capstone, or in the CMV app. There also could have been error in calculations and compensating for the tape measure being mounted higher than the drop because of the force meter.

If we could do the experiment differently, I would mount the tape measure on the same bar that the string was tied to and move the force meter out of the way. I would also keep length constant in all trials to achieve simpler and more precise results.

Conclusion

Through this experiment we found a relationship between $h$ and $k$ that will be beneficial as we move forward in the challenge. We should be able to use this relationship to extrapolate the data and determine the variables needed for the actual drop. Our next step could be to run more trials to ensure accuracy in our results. We also could try varying the mass and observing its effect on the relationships.